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Helicity Probabilities For Heavy Quark Fragmentation Into Excited Mesons

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Abstract

In the fragmentation of a heavy quark into a heavy meson whose light degrees of freedom have angular momentum $3/2$, all the helicity probabilities are completely determined in the heavy quark limit up to a single probability $w_{3/2}$. We point out that this probability depends on the longitudinal momentum fraction z of the meson and on its transverse momentum p_{\perp} relative to the jet axis. We calculate $w_{3/2}$ as a function of scaling variables corresponding to z and p_{\perp} for the heavy quark limit of the perturbative QCD fragmentation functions for b quark to fragment into $(b\bar{c})$ mesons. In this model, the light degrees of freedom prefer to have their angular momentum aligned transverse to, rather than along, the jet axis. Implications for the production of excited heavy mesons, like D^{**} and B^{**} , are discussed.

The discovery of the powerful heavy quark spin-flavor symmetry [1] in Quantum Chromodynamics (QCD) and the development of the Heavy Quark Effective Theory (HQET) [2] have greatly improved our theoretical understanding of hadrons containing a single heavy quark Q . The crucial idea in HQET is that both the heavy quark mass and spin decouple from the strong interaction dynamics between the heavy quark and the light degrees of freedom in the limit of infinitely heavy quark mass. The decouplings of the mass and spin occur because in the limit $m_Q \rightarrow \infty$ the only variable to describe a free propagating heavy quark is its 4-velocity v . In the presence of gauge fields, the propagation of the heavy quark is simply described by the Wilson line $P \exp i \int A_\mu v^\mu dt$, which contains no information about the heavy quark flavor nor its spin. Furthermore, the leading operator – the chromo-magnetic dipole moment that couples heavy quark spin to the gluon field is inversely proportional to the heavy quark mass. To the first approximation in the heavy quark mass expansion, a heavy quark can be treated as a static color source for the remaining light degrees of freedom that make up the physical heavy-light hadrons observed in Nature.

Falk and Peskin [3] have recently pointed out that heavy quark spin symmetry provides strong constraints on the helicity probabilities of the heavy mesons produced by the fragmentation/hadronization of a heavy quark. For heavy mesons whose light degrees of freedom have angular momentum $1/2$, the helicity probabilities are completely determined in the heavy quark limit. If the light degrees of freedom have angular momentum $3/2$, the probabilities are determined up to a single parameter $w_{3/2}$. In this paper, we point out that the probability $w_{3/2}$ depends on the longitudinal momentum fraction z of the heavy meson relative to the heavy quark jet and its transverse momentum p_\perp relative to the jet axis. We calculate $w_{3/2}$ as a function of the scaling variables corresponding to z and p_\perp for the $m_b \rightarrow \infty$ limit of the perturbative QCD (PQCD) fragmentation functions for a b quark to fragment into $(b\bar{c})$ mesons. These fragmentation functions can be used as a model for the fragmentation of a heavy quark into heavy-light mesons. The implications for the production of excited heavy-light mesons, like the D^{**} and B^{**} , are discussed.

In the heavy quark limit it is convenient to label hadronic states by the eigenvalues j and j_l of the total angular momentum \vec{J} of the hadron and the angular momentum \vec{J}_l of the light degrees of freedom respectively [4]. In general, the spectrum of hadrons containing

a single heavy quark Q has, for each j_l , a degenerate doublet (j_-, j_+) with total angular momentum $j_\pm = j_l \pm 1/2$. (For the exceptional case of a baryon with $j_l = 0$, there is only a singlet with total angular momentum $1/2$.) For the heavy-light $(Q\bar{q})$ mesons, one can write $\vec{J}_l = \vec{S}_q + \vec{L}$ where \vec{S}_q is the spin of the valence light antiquark \bar{q} and \vec{L} is the orbital angular momentum with integer eigenvalue L . For S -wave heavy-light mesons, $L = 0$ and $j_l^P = \frac{1}{2}^-$, we have the familiar degenerate doublet $(j_-^P, j_+^P) = (0^-, 1^-)$ consisting of a pseudoscalar and a vector meson. For P -wave heavy-light mesons, $L = 1$ and the j_l^P of the light degrees of freedom can be either $\frac{1}{2}^+$ or $\frac{3}{2}^+$. In this case, we have two distinct doublets $(j_-^P, j_+^P) = (0^+, 1^{+'})$ and $(1^+, 2^+)$ for $j_l^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ respectively. While the 0^+ and 2^+ states are identified as the 3P_0 and 3P_2 states constructed in the LS coupling scheme respectively, the $1^{+'}$ and 1^+ states are linear combinations of the 1P_1 and 3P_1 states: $|1^{+'}\rangle = \sqrt{1/3}|^1P_1\rangle + \sqrt{2/3}|^3P_1\rangle$ and $|1^+\rangle = -\sqrt{2/3}|^1P_1\rangle + \sqrt{1/3}|^3P_1\rangle$.

Falk and Peskin [3] showed that heavy quark spin symmetry combined with the parity invariance of QCD interactions can impose very useful relations among the probabilities for a heavy quark with a given helicity to fragment/hadronize into the various helicity states of the heavy-light hadrons within the same doublet. For definiteness, we assume the heavy quark Q is purely left-handed in what follows. For the case of $j_l = \frac{1}{2}$, parity invariance implies that the two fragmentation probabilities for the heavy quark Q to hadronize by combining with light degrees of freedom with helicity $m_l = -1/2$ or $+1/2$ must be the same. Although heavy quark symmetry allows coherent superposition of the two zero helicities states in the corresponding $(0, 1)$ doublet at the very early stage of the fragmentation process, the relative population of the four helicities states in the doublet are completely determined by symmetry. The resulting table of fragmentation probabilities is [3]

$$\begin{pmatrix} P_1(h) \\ P_0(h) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ & & \frac{1}{4} \end{pmatrix}, \quad (1)$$

where the helicity h runs through the values $-1, 0, +1$ across the table. However, for $j_l = \frac{3}{2}$, parity invariance implies the following table of fragmentation probabilities for a heavy quark

to fragment into various helicity states of the light degrees of freedom [3]

$$P_{3/2}(m_l) = \left(\frac{1}{2}w_{3/2}, \frac{1}{2}(1 - w_{3/2}), \frac{1}{2}(1 - w_{3/2}), \frac{1}{2}w_{3/2} \right) , \quad (2)$$

where the helicity m_l of the light degrees of freedom runs through the values $-\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}$ across the table. The Falk-Peskin parameter, $w_{3/2}$, is the conditional probability for the heavy quark to fragment into a meson system whose light degrees of freedom are in the maximum helicity states $m_l = \pm 3/2$. It can take values between 0 and 1. At the very early stage in the fragmentation process, heavy quark symmetry allows coherent linear superposition of the various components in the $(1, 2)$ doublet which have the same helicity $h = m_l - \frac{1}{2}$. However, at a later time determined by the mass splittings within the doublet, the various helicity components of the $(1, 2)$ doublet will propagate incoherently, resulting in the following table of fragmentation probabilities [3]

$$\begin{pmatrix} P_2(h) \\ P_1(h) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}w_{3/2} & \frac{3}{8}(1 - w_{3/2}) & \frac{1}{4}(1 - w_{3/2}) & \frac{1}{8}w_{3/2} & 0 \\ & \frac{1}{8}(1 - w_{3/2}) & \frac{1}{4}(1 - w_{3/2}) & \frac{3}{8}w_{3/2} & \end{pmatrix} , \quad (3)$$

with the helicity h runs through the values $-2, -1, 0, +1, +2$ across the table. Therefore, all the relative fragmentation probabilities for the 8 helicity states of a $(1, 2)$ doublet are determined, up to the Falk-Peskin parameter $w_{3/2}$. This non-perturbative parameter can not be determined without a dynamical calculation.

Consider the leading order Feynman diagram shown in Fig.1 for a heavy b quark to fragment into a heavy-heavy ($b\bar{c}$) meson by creating a $c\bar{c}$ pair from the vacuum. In the limit of $m_b/m_c \rightarrow \infty$, this can be taken as a model for heavy quark fragmentation into heavy-light mesons. Following our previous works in [5, 6, 7] (see also [8]), the fragmentation function $D(z)$ can be expressed as $D(z) = \int_{s_0(z)}^{\infty} ds D(z, s)$, with $s_0(z) = M^2/z + r^2 M^2/(1 - z)$ and $D(z, s) = (16\pi^2)^{-1} \lim_{q_0/m_b \rightarrow \infty} (|\mathcal{M}|^2/|\mathcal{M}_0|^2)$. \mathcal{M} is the amplitude for a high energy source (symbolically denoted by Γ in Fig.1) to create $b^*(q) \rightarrow (b\bar{c})(p) + c(p')$ with total 4-momentum $q = p + p'$, and \mathcal{M}_0 denotes the same source creating an on-shell heavy b quark with the same 3-momentum \vec{q} . In the nonrelativistic approximation, $M = m_b + m_c$; and we define $r = m_c/M$. $s = q^2$ is the virtuality of the fragmenting b quark. In a frame where the virtual

b^* quark has 4-momentum $q = (q_0, 0, 0, q_3)$, the longitudinal momentum fraction of $(b\bar{c})$ is $z = (p_0 + p_3)/(q_0 + q_3)$ and its transverse momentum is $\vec{p}_\perp = (p_1, p_2)$. For the b quark propagator and bbg vertex that entered in Fig.1, we will apply the techniques of HQET [2]. It is straightforward to write down the following matrix elements $\mathcal{M}(^1P_1, h)$ and $\mathcal{M}(^3P_1, h)$ for $b \rightarrow (b\bar{c})$, with the $(b\bar{c})$ bound state in definite helicity h of the 1P_1 and 3P_1 states respectively:

$$\mathcal{M}(^1P_1, h) = i\delta_{ij}g^2C_F R'(0)\sqrt{\frac{3}{16r^2\pi N_c M}}\frac{1}{k^4 v \cdot k}\epsilon_\alpha^*(h)\bar{u}(p')\gamma_5 V^\alpha \left(\frac{1+\not{p}}{2}\right)\Gamma, \quad (4)$$

with the vertex V^α given by

$$V^\alpha = 4rMq^\alpha\left(1 - \frac{v \cdot k}{n \cdot k}\not{n}\right) - 2rM\frac{k^2 v \cdot k}{(n \cdot k)^2}n^\alpha \not{n} + k^2\left(1 + \frac{v \cdot k}{n \cdot k}\not{n}\right)\gamma^\alpha; \quad (5)$$

and

$$\mathcal{M}(^3P_1, h) = -\delta_{ij}g^2C_F R'(0)\sqrt{\frac{3}{32r^2\pi N_c M}}\frac{1}{k^4 v \cdot k}\epsilon_{\alpha\beta\mu\nu}v^\mu\epsilon^{\nu*}(h)\bar{u}(p')V^{\alpha\beta}\left(\frac{1+\not{p}}{2}\right)\Gamma, \quad (6)$$

with the vertex tensor $V^{\alpha\beta} = \tilde{V}^\alpha\gamma^\beta$, where \tilde{V}^α can be obtained from V^α in (5) with $\not{n} \rightarrow -\not{n}$ and $\gamma^\alpha \rightarrow -\gamma^\alpha$. In (4) and (6), i and j are the fundamental color indices, N_c is the number of color, $C_F = (N_c^2 - 1)/(2N_c)$, and $R'(0)$ and $\epsilon(h)$ are the derivative of the radial wave function and the helicity wave function for the bound state respectively. We have picked the axial gauge associated with the vector $n^\mu = (1, -\vec{p}/|\vec{p}|)$. In this gauge, the short-distance process that creates the energetic heavy quark b^* (symbolically denoted by Γ in Fig.1) and the subsequent fragmentation of the heavy quark become manifestly factorized. In the above equations, we have used $p = Mv$ and $q = m_b v + k$, where v and k are the 4-velocity and residual momentum of the b quark respectively. Some useful kinematical relations are $k^2 = 2rv \cdot k = r(s - (1-r)^2 M^2)/M$. The tree-level matrix element squared $|\mathcal{M}_0|^2$ is given by $N_c(M/z)\text{Tr}[\Gamma\bar{\Gamma}(1 + \not{p})]$. In writing down the above matrix elements, we have assumed the P -wave bound state is a color-singlet. It has been pointed out recently by Bodwin-Braaten-Lepage [9] that, beyond leading orders in the calculations of the production and decay rates of P -wave quarkonium [9, 10] and $(b\bar{c})$ mesons [7], there is also a color-octet

S -wave mechanism needed to be taken into account in order to avoid infrared divergencies that spoil factorization. We will also include the color-octet S -wave contributions in what follows.

Heavy quark fragmentation functions have nontrivial limits as $m_Q \rightarrow \infty$ (or $r \rightarrow 0$) if they are expressed in terms of the scaling variable $y = (1/z - 1 + r)/r$, first used by Jaffe and Randall [11], and the rescaled transverse momentum $t = |\vec{p}_\perp|/(rM)$. We therefore define the fragmentation functions $D(y, t)$, $D(t)$, and $D(y)$ according to the following changes of variables:

$$\int_0^1 dz D(z) = \int_0^1 dz \int_{s_0(z)}^\infty ds D(z, s), \quad (7)$$

$$= \int_1^\infty dy \int_0^\infty dt D(y, t), \quad (8)$$

$$= \int_0^\infty dt D(t), \quad (9)$$

$$= \int_1^\infty dy D(y). \quad (10)$$

The relation among s , t , and z is given by $s = M^2[(1 + r^2 t^2)/z + r^2(1 + t^2)/(1 - z)]$. Following the same procedures as in Refs.[5, 6, 7], we square the matrix elements $\mathcal{M}(^1P_1, h)$ and $\mathcal{M}(^3P_1, h)$, calculate the interference between these two amplitudes, and project out the transversely and longitudinally polarized 1^+ states, one can deduce the generalized polarized fragmentation functions $D_T(y, t) = D_T^{(1)}(y, t) + 2 D^{(8)}(y, t)$ and $D_L(y, t) = D_L^{(1)}(y, t) + D^{(8)}(y, t)$, depending on both variables y and t . In the limit $r \rightarrow 0$, the color-singlet pieces $D_{T,L}^{(1)}(y, t)$ are given by

$$D_T^{(1)}(y, t) = \frac{4C_1}{3} \frac{(y-1)^2 t}{y^4(t^2 + y^2)^6} \left\{ (y-4)^2 y^6 + y^4(40 - 32y + 26y^2 - 4y^3 + y^4)t^2 \right. \\ \left. + 3y^3(16 - 8y + 2y^2 + 3y^3)t^4 + 3y^2(8 - 4y + 3y^2)t^6 + (1+y)^2 t^8 \right\}, \quad (11)$$

$$D_L^{(1)}(y, t) = \frac{8C_1}{3} \frac{(y-1)^2 t}{y^4(t^2 + y^2)^6} \left\{ (y-4)^2 y^6 + y^4(4 + 40y - 10y^2 - 4y^3 + y^4)t^2 \right. \\ \left. + 3y^3(4 + 13y - 4y^2)t^4 + 3y^2(5 + 2y)t^6 + (1+y)^2 t^8 \right\}, \quad (12)$$

where $C_1 = 2\alpha_s^2 |R'(0)|^2 / (3\pi N_c m_c^5)$. The color-octet piece $D^{(8)}(y, t)$ can be extracted from Ref.[12] by taking the limit $r \rightarrow 0$,

$$D^{(8)}(y, t) = 12C_8 \frac{(y-1)^2 t}{y^2(t^2 + y^2)^4} [4y^2 + y(y+4)t^2 + t^4] , \quad (13)$$

with $C_8 = 3\alpha_s^2 H'_8 / (256 N_c m_c)$ where H'_8 is a nonperturbative parameter associated with the color-octet mechanism for P -wave production [7, 9, 10]. We will treat the overall constants C_1 and C_8 as free parameters in our approach. One notices that in terms of the variables y and t , the leading order results of the fragmentation functions $D_{T,L}(y, t)$ scale, *i.e.* they do not depend on the heavy quark mass.

This motivates the introduction of the generalized Falk-Peskin parameter $w_{3/2}(y, t)$ that is a function of the two scaling variables y and t . The generalized polarized fragmentation functions $D(y, t)$ for the 8 helicity components of a $(1, 2)$ doublet satisfy a similar table like that given in (3) with $w_{3/2}$ replaced by $w_{3/2}(y, t)$. From the probability table for $P_1(h)$ in (3), we see that D_T is proportional to $(1 - w_{3/2})/8 + 3w_{3/2}/8$, while D_L is proportional to $(1 - w_{3/2})/4$. Thus $w_{3/2}(y, t)$ can be defined in terms of the fragmentation functions for the spin-1 state by

$$w_{3/2}(y, t) = \frac{D_T(y, t) - \frac{1}{2}D_L(y, t)}{D_T(y, t) + D_L(y, t)} . \quad (14)$$

Simple analytic expression for $w_{3/2}(y, t)$ can be obtained from (11)–(13) as

$$w_{3/2}(y, t) = \frac{n(y, t) + \epsilon h(y, t)}{d(y, t) + 2\epsilon h(y, t)} , \quad (15)$$

with $n(y, t) = 6y^2(y-1)^2 t^2 [4y^2 + y(y+4)t^2 + t^4]$, $d(y, t) = 2(t^2 + y^2) [y^4(y-4)^2 + y^3(24 + y - 4y^2 + y^3)t^2 + y^2(17 - 2y + 2y^2)t^4 + (1 + y)^2 t^6]$, $h(y, t) = 9y^2(t^2 + y^2)^2 [4y^2 + y(y+4)t^2 + t^4]$, and $\epsilon = C_8/C_1$.

We can define probabilities $w_{3/2}(t)$ and $w_{3/2}(y)$ that depend only on a single scaling variable by an expression analogous to (14), except with $D_{T,L}(y, t)$ replaced by $D_{T,L}(t)$ and $D_{T,L}(y)$ respectively. Integrating the fragmentation functions in (11)–(13) over y and forming

a similar ratio like (14), we obtain $w_{3/2}(t)$:

$$w_{3/2}(t) = \frac{9}{80} \left(\frac{n_1 + n_2 \arctan(t) + n_3 \log(1 + t^2)}{d_1 + d_2 \arctan(t) + d_3 \log(1 + t^2)} \right), \quad (16)$$

with $n_1 = t[630 - 5(521 - 240\epsilon)t^2 + (231 - 2440\epsilon)t^4]$, $n_2 = -5[126 - 15(49 - 16\epsilon)t^2 + 8(20 - 99\epsilon)t^4 + (5 + 72\epsilon)t^6]$, $n_3 = -320t[4 - 2(2 - 3\epsilon)t^2 - 3\epsilon t^4]$, and $d_1 = t[105 - 2(406 - 135\epsilon)t^2 + (79 - 549\epsilon)t^4]$, $d_2 = -3[35 - 5(71 - 18\epsilon)t^2 + 3(31 - 99\epsilon)t^4 + 3(1 + 9\epsilon)t^6]$, $d_3 = -72t[4 - 6(1 - \epsilon)t^2 - 3\epsilon t^4]$. The curve of $w_{3/2}(t)$ versus t is plotted in Fig.2 for the two cases of $\epsilon = 0$ (color-singlet dominance) and $\epsilon = \infty$ (color-octet dominance).

Similarly, by integrating the fragmentation functions in (11)–(13) over t and forming the ratio like (14), we obtain $w_{3/2}(y)$:

$$w_{3/2}(y) = \frac{1}{10} \frac{(y-1)^2(12 + 8y + 5y^2) + 15\epsilon y^2(8 + 4y + 3y^2)}{(8 + 4y^2 + y^4) + 3\epsilon y^2(8 + 4y + 3y^2)}. \quad (17)$$

The curve of $w_{3/2}(y)$ versus y is plotted in Fig.3 for the two cases of $\epsilon = 0$ and ∞ .

The original Falk-Peskin parameter $w_{3/2}$ is given by (14), with the numerator and denominator integrated over both y and t . The total fragmentation probabilities are $\int_1^\infty dy \int_0^\infty dt D_T(y, t) = (86/315)C_1 + (16/5)C_8$ and $\int_1^\infty dy \int_0^\infty dt D_L(y, t) = (17/63)C_1 + (8/5)C_8$. These imply

$$w_{3/2} = \frac{1}{6} \left(\frac{29 + 504\epsilon}{19 + 168\epsilon} \right). \quad (18)$$

We note that setting $\epsilon = 0$ in (18) gives $w_{3/2} \rightarrow 29/114$, a result first derived by Chen and Wise [13]. While the Falk-Peskin parameter $w_{3/2}$ is independent of QCD evolution, its generalizations $w_{3/2}(y, t)$, $w_{3/2}(t)$, and $w_{3/2}(y)$ given above are evaluated at a subtraction point around the heavy quark mass scale. Their values at a higher scale can be determined by the usual Altarelli-Parisi evolution of the fragmentation functions.

The Falk-Peskin parameter $w_{3/2}$ determines all the helicity probabilities for the fragmentation of a heavy quark into a heavy meson whose light degrees of freedom have angular

momentum $3/2$. In this Letter, we have pointed out that $w_{3/2}$ can have nontrivial dependence on the scaling variables y and t corresponding to the longitudinal momentum fraction z and the transverse momentum p_\perp of the meson relative to the heavy quark jet. We have calculated $w_{3/2}$ as a function of y and t for the $m_b \rightarrow \infty$ limit of the perturbative QCD fragmentation functions for the production of P -wave ($b\bar{c}$) mesons. We find that the probability $w_{3/2}$ has significant dependence on y and t , varying over the range from 0 to $1/2$. The PQCD fragmentation functions can be used as a model for the fragmentation of a heavy quark into heavy-light mesons, and applied to the fragmentation processes $c \rightarrow D^{**}$ and $b \rightarrow B^{**}$. The prediction $w_{3/2} \leq 1/2$ of this model implies that light degrees of freedom with helicity $m_l = \pm 1/2$ always have a larger population than the maximum helicity states of $m_l = \pm 3/2$. This prediction supports the speculation of Falk and Peskin [3] that the angular momentum of the light degrees of freedom with $j_l = 3/2$ prefers to align transverse to, rather than along, the fragmentation axis. This spin alignment of the heavy quark can be detected by measuring the anisotropies of the decay products from the hadronic transitions between the two doublets $(1^+, 2^+)$ and $(0^-, 1^-)$ [3]. The PQCD fragmentation model predicts that these anisotropies vary significantly with y and t . In the case of charm quark fragmentation into D^{**} , an upper bound of $w_{3/2} \leq 0.25$ at the 90% confidence level has been deduced from the existing experimental data [3]. After integrating over y and t , the prediction (18) of the PQCD fragmentation model with $\epsilon = 0$ is $w_{3/2} \approx 0.25$, which suggests that present experiments may be close to observing a nonzero value for $w_{3/2}$. String models of fragmentation tend to give significantly larger values of $w_{3/2}$ and may already have been ruled out [3]. We conclude that future measurements of the Falk-Peskin probability $w_{3/2}$ for the charm and bottom systems, including the dependence of $w_{3/2}$ on the scaling variables y and t , can provide valuable insights into the dynamics of heavy quark fragmentation.

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Figure Captions

1. Leading order diagram for the heavy b quark to fragment into a P -wave ($b\bar{c}$) bound state as a model for heavy quark fragmentation into excited heavy-light meson. The

outgoing momenta are $(1-r)p + \delta$, $rp - \delta$, and p' for the b , \bar{c} , and c , respectively. δ is the relative momentum of the b and \bar{c} .

2. $w_{3/2}(t)$ versus the rescaled transverse momentum t at the heavy quark mass scale.
3. $w_{3/2}(y)$ versus the scaling variable y at the heavy quark mass scale.

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